Destination L1: A Thematic Unit

Studying Orbits About Bodies in Space

STUDENT ACTIVITY

BACKGROUND INFORMATION

Using a surprising simple relationship, Kepler's Third Law, we can study the orbit of a satellite about some body in space. We can study the basic parameters of (1) the time our satellite will take to circle the body once (called the "Period") and (2) its average distance from the body (we will call this "A"). Kepler discovered this in 1619, after having spent the previous 19 years studying the best data available, at that time, of the orbit of Mars. This equation is empirical. That is, it was not based on theory, but the very careful analysis of the data.

Assume we have a body in space, say the Earth, and we have a satellite in orbit about it with an average altitude above the surface of "a" (in whatever units you like—miles, km, etc.). We will assume also our body is a sphere with a radius of "R." "F" is the center of the body. Then, the total distance from the center of our body to the satellite is (a+R). (The radius of the body plus the altitude our satellite is above its surface.) Let us label this total distance, a+R, with the letter "A". Finally, the time it takes our satellite to go around the body exactly one time is call the "Period" of the orbit, or we will say just "P."
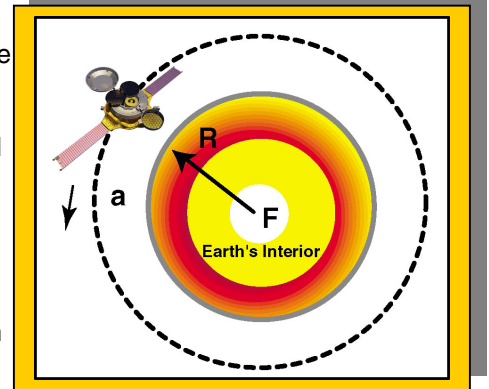
What did Kepler discover about this kind of a "two-body system?" He found from his careful analysis of the data that:

$$P^2 \text{ is proportional to } A^3$$

or in the form of an equation:

$$P^2 = k \times A^3 \qquad \text{or } k = P^2/A^3$$

With some work, we could figure out this constant. But if we set this equation up as a ratio, we do not need to know the constant *(*k). In addition, as long as we are consistent with our units, we can use any we like and the ratio equation does not change. That is,

$$\frac{P^2_1}{P^2_2} = \frac{A^3_1}{A^3_2} \qquad \text{(Equation1)}$$

What good does this ratio equation do us? If we know the information about any one orbit about a particular body, we can figure out the information about any other orbit. Where can we get the information we need about an orbit? By looking at the following table of special "imaginary" orbits. That is, we will imagine that our body in space is a perfect sphere, is completely smooth, and has no atmosphere. We can even "hold our breaths for a moment" and imagine this also about the Earth. Now what would be the period for a satellite to go around our body once, when it is just "skimming the surface," that is, the total distance from the center of the body to our satellite is just the radius of the body R.

USING KEPLER'S THIRD LAW

TABLE OF PERIODS FOR ORBITS OF SOME BODIES IN SPACE

Body	Period (minutes)	Radius of the Body [R] Kilometers	Miles
Mercury	85.1	2439	1534
Venus	87.8	6052	3761
Earth	84.7	6378	3964
Mars	100.7	3340	2075
Jupiter	174.0	71,434	44,397
Saturn	253.4	60,272	37,459
Uranus	156.4	25,575	15,895
Neptune	155.9	24,747	15,380
Pluto	208.4	1150	715

To show how easy it will now be to determine the period and average altitude of a satellite about one of the bodies in our table, let us look at an Earth satellite at an average altitude of 150 miles, or about 240 km in metric units, and ask the question, "what is the period of the satellite?" In place of the subscripts of "1" and "2" in equation (1), let us use "R" for reference in place of "1," and "S" for satellite in place of "2".

After you cross-multiply and solve for P^2_s, we get:

$$P^2_S = (P^2_R A^3_S) / A^3_R \qquad \text{(Equation 2)}$$

Using the reference orbit for Earth and the numbers of our problem, we get:

$$P^2_S = (84.70 \text{ minutes})^2 \text{ x } (3964.00 \text{ miles} + 150.00 \text{ miles})^3 / (3964.00 \text{ miles})^3$$

Note that the 4114 value comes from adding the radius of the Earth, 3964 miles, and the altitude of the spacecraft, 150 miles.

$$P^2_S = (84.70 \text{ minutes})^2 \text{ x } (4114.00 \text{ miles})^3 / (3964.00 \text{ miles})^3$$

We can use a hand calculator (or paper and pencil) to multiply out the numbers to get:

$$P^2_S = (7174.09 \text{ minutes}^2 \text{ x } 6.96 \text{ x } 10^{10}) / 6.23 \text{ x } 10^{10} \text{miles}^3$$

$$P^2_S = 4.99 \text{ x } 10^{14} \text{ minutes}^2 \text{miles}^3 / 6.23 \text{ x } 10^{10} \text{miles}^3$$

$$P^2_S = 8014.71 \text{ minutes}^2$$

$$P_s = 89.53 \text{ minutes}$$

It might surprise you that the period is only a little under five minutes longer than our reference orbit at the surface of the Earth. You can now use the Excel workbook, Kepler3rd.xls, to examine other Earth orbits or orbits about any of the bodies in our table.

PROCEDURE

Using the formulas shown above, find the orbital period of a satellite that is 150 miles above the planet Mars. Use the space below or your own paper to show your work.

TECHNOLOGY APPLICATION

1. The Kepler3rd.xls spreadsheet will calculate a table of 100 orbits beginning with an orbit in the first row of column "C," and then steps each calculation outward by a value defined in the first row of column "B," (delta A). If you choose to "check" the work on the Mars satellite, plug in the numbers in parenthesis.

2. Open the *Kepler's Third Excel Calculation Using Macro.* The spreadsheet will come up with the heading in each column.

3. In the first row of column "A," "Body Radius," enter the radius of the body you plan to orbit—use any units you wish, but once you choose, make sure you are consistent with your other distance entries. (2075.00) This number represents the radius of Mars.

4. In the first row of column "B," "Delta A," (100.00) This number represents the distance between each of the one hundred orbits. This can be an arbitrary number.

5. In the first row of column "C," enter the correct period for the body you have chosen. (100.70)

6. In the first row of column "D," enter the value of the radius of the chosen body added to the lowest altitude of your orbits. (2225.00) This number is the period plus the altitude of the satellite. Click anywhere on the spreadsheet.

7. Using the "Tools" pull down menu, choose "Macro" and then "Macros."

8. Click "Run" and *presto*, you have your table of values!

9. Complete the questions on the student reporting sheet.